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A SUMMATION (PARTIAL SUM) OF HARMONIC PROGRESSION SERIES PRODUCED THE SUM EXIST

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ABSTRACT

The sum up-to n terms of Arithmetic Progression is expressed by

- S=n/2[2a+(n-1)d] and
- S=n/2(a+l)

Similarly, we also have formula for sum up to n terms of Geometric Progression for r>1 and r<1.But we have no formula for sum up-to n terms of Harmonic Progression. In this section we have an expression for getting sum up-to definite terms (as n terms) of Harmonic Progression.

KEYWORDS: Partial Sum, Order of n, Just Middle Term, Two Halves, Variable, Approximation

INTRODUCTION

If partial sum of $\frac{1}{a}+\frac{1}{a+d}+\frac{1}{a+2d}+\ldots+\frac{1}{a+(n-1)d}+\ldots\ldots\ldots$ is

$\frac{1}{a}+\frac{1}{a+d}+\frac{1}{a+2d}+\ldots+\frac{1}{a+(n-1)d}$

then

$$S_n=\mu\left(\frac{3A^2-D^2}{aAl}\right)+\frac{n-3}{2}\left[\frac{1}{a+(\frac{n-1}{4})d}+\frac{1}{l-(\frac{n-1}{4})d}\right]$$

$$\text{Where } \mu=\begin{cases} 1+\frac{3n}{200}\ if\ n<25 \\ 1+\frac{n}{50}\ if 25\le n<100 \\ 2+\frac{3n}{500}\ if\ 100\le n\le 200 \\ 3+(\frac{n}{440})^{\frac{1}{o(n)-2}}\ if\ n>200 \end{cases}$$

a,d,n and l are as usual meaning while

A= $a+(\frac{n-1}{2})d$

D=A-a, μ is a constant, and∘(n) is order of n.

Note:-If Progression Starts from 1(or more than 1) and d be more than 2 then leave first term and evaluate sum of rest terms and then add first term to it. If d=2 then apply condition of $n>200$ for any value of n.

Proof:-

Let H.P. is

$\frac{1}{a}+\frac{1}{a+d}+\frac{1}{a+2d}+\ldots..+\frac{1}{a+(n-1)d}+\ldots\ldots\ldots\ldots$

Its partial sum be $\frac{1}{a}+\frac{1}{a+d}+\frac{1}{a+2d}+\ldots..+\frac{1}{a+(n-1)d}$

Now, we select

$\frac{1}{a},\frac{1}{a+\frac{(n-1)d}{2}}$=just middle term and $\frac{1}{a+(n-1)d}$ such as their sum produced by S_1.

Let

$\frac{\mathbf{1}}{\boldsymbol{a}}=\frac{1}{a},\frac{\mathbf{1}}{\boldsymbol{a+d}}=\frac{1}{a+\frac{(n-1)d}{2}},\frac{\mathbf{1}}{\boldsymbol{a+2d}}=\frac{1}{a+(n-1)d}$

Now adding them, we have

$S_1=\frac{1}{a}+\frac{1}{a+d}+\frac{1}{a+2d}$

$=\frac{a^2+3ad+2d^2+a^2+2ad+a^2+ad}{a(a+d)(a+2d)}$

$=\frac{3a^2+6ad+2d^2}{a(a+d)(a+2d)}$

$=\frac{3(a^2+2ad+d^2)-d^2}{a(a+d)(a+2d)}$

$=\frac{3(a+d)^2-d^2}{a(a+d)(a+2d)}=\frac{3A^2-D^2}{aAl}$ (1)

It doesn't mean d=D, because here in (1) only taken three terms and obviously in this case d becomes equal to D. But in general d≠D.

Now, we have to evaluate S_2' because

$S=S_1+S_2$ (2)

S is whole sum and S_2 is sum of (n-3) terms

On breaking it into two equal parts, it has two halves containing $\frac{n-3}{2}$ terms in each half.

Now,

$S_2=\left(\frac{n-3}{2}\right)terms\ +\ \left(\frac{n-3}{2}\right)terms$

↓ ↓

First half second half

So. After analysis, we get

$$S_2=\frac{n-3}{2}\left[\frac{1}{a+(\frac{n-1}{4})d}+\frac{1}{l-(\frac{n-1}{4})d}\right] \qquad (3)$$

Here $\frac{1}{a+(\frac{n-1}{4})d}$ & $\frac{1}{l-(\frac{n-1}{4})d}$ are terms just middle terms of both halves (first and second) respectively even series consist even terms (in this case these terms are numerically same as should term if possible)

Thus by (A), $S=S_1+S_2$

$$\Rightarrow S=\left(\frac{3A^2-D^2}{aAl}\right)+\frac{n-3}{2}\left[\frac{1}{a+(\frac{n-1}{4})d}+\frac{1}{l-(\frac{n-1}{4})d}\right]$$

Further, by analytic way, for reducing approximation (some numeric error) in this formula. I introduce a variable μ which depends on numbers of terms and it reduces this approximation such a way that

$$S_n=\mu\left(\frac{3A^2-D^2}{aAl}\right)+\frac{n-3}{2}\left[\frac{1}{a+\left(\frac{n-1}{4}\right)d}+\frac{1}{l-\left(\frac{n-1}{4}\right)d}\right]$$

CONCLUSIONS

This gives a way to calculate sum up-to n terms of Harmonic Progression by which we can calculate sum up-to desired terms same as Arithmetic Progression and Geometric Progression.

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